$\overrightarrow{a}\ /\!/\ \overrightarrow{b}$)

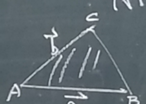
$\Rightarrow \overrightarrow{a}\times\overrightarrow{b}$ 為 $\overrightarrow{a}$ 與 $\overrightarrow{b}$ 的公垂向量

$\overrightarrow{a}=(a_1\ a_2\ a_3)\quad \overrightarrow{b}=(b_1\ b_2\ b_3)$

$\overrightarrow{a}\cdot\overrightarrow{b}=|\overrightarrow{a}||\overrightarrow{b}|\cos\theta$.

$$|\overrightarrow{a}\times\overrightarrow{b}|=\left|\left(\begin{vmatrix}a_2 & a_3\\ b_2 & b_3\end{vmatrix},\begin{vmatrix}a_3 & a_1\\ b_3 & b_1\end{vmatrix},\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}\right)\right|$$

$$=\sqrt{\begin{vmatrix}a_2 & a_3\\ b_2 & b_3\end{vmatrix}^2+\begin{vmatrix}a_3 & a_1\\ b_3 & b_1\end{vmatrix}^2+\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}^2}$$



$$\triangle ABC=\frac{1}{2}\sqrt{|\overrightarrow{a}|^2|\overrightarrow{b}|^2-(\overrightarrow{a}\cdot\overrightarrow{b})^2}.$$

$$=\frac{1}{2}\sqrt{(a_1^2+a_2^2+a_3^2)(b_1^2+b_2^2+b_3^2)-(a_1b_1+a_2b_2+a_3b_3)^2}$$

補充

$$\triangle ABC=\frac{1}{2}|\overrightarrow{a}|(|\overrightarrow{b}|\sin\theta)$$

$$=\frac{1}{2}|\overrightarrow{a}||\overrightarrow{b}|\sin\theta$$

$$=\frac{1}{2}|\overrightarrow{a}||\overrightarrow{b}|\sqrt{1-\cos^2\theta}$$

$$=\frac{1}{2}\sqrt{|\overrightarrow{a}|^2|\overrightarrow{b}|^2-|\overrightarrow{a}|^2|\overrightarrow{b}|^2\cos^2\theta}$$

$$=\frac{1}{2}\sqrt{|\overrightarrow{a}|^2|\overrightarrow{b}|^2-(\overrightarrow{a}\cdot\overrightarrow{b})^2}$$

$$=\frac{1}{2}\sqrt{\cancel{a_1^2b_1^2}+a_1^2b_2^2+a_1^2b_3^2+a_2^2b_1^2+\cancel{a_2^2b_2^2}+a_2^2b_3^2+a_3^2b_1^2+a_3^2b_2^2+\cancel{a_3^2b_3^2}-(\cancel{a_1^2b_1^2}+\cancel{a_2^2b_2^2}+\cancel{a_3^2b_3^2}+2a_1a_2b_1b_2+2a_1b_1a_3b_3+2a_2b_2a_3b_3)}$$

$$=\frac{1}{2}\sqrt{(a_1b_2-a_2b_1)^2+(a_1b_3-a_3b_1)^2+(a_3b_1-a_1b_3)^2}$$

$$=\frac{1}{2}\sqrt{\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix}^2+\begin{vmatrix}a_1 & a_3\\ b_1 & b_3\end{vmatrix}^2+\begin{vmatrix}a_1 & a_3\\ b_1 & b_3\end{vmatrix}^2}$$

$$=\frac{1}{2}|\overrightarrow{a}\times\overrightarrow{b}|$$

$\vec{a}=(a_1, a_2, a_3)$　$\vec{b}=(b_1, b_2, b_3)$，$\vec{c}=(c_1, c_2, c_3)$

$\Rightarrow$ $\vec{a}, \vec{b}, \vec{c}$ 所構成的平行六面體的體積

為 $\left|\vec{a}\cdot(\vec{b}\times\vec{c})\right| = \left|\vec{b}\cdot(\vec{a}\times\vec{c})\right| = \left|\vec{c}\cdot(\vec{a}\times\vec{b})\right|$

(对话式 P.50)　　设 $\vec{n} = \vec{a}\times\vec{b}$

V = 底面積 × 高

$= \left|\vec{a}\times\vec{b}\right| \times \left|\dfrac{\vec{c}\cdot\vec{n}}{|\vec{n}|}\right|$

$= \left|\vec{c}\cdot\vec{n}\right| = \left|\vec{c}\cdot(\vec{a}\times\vec{b})\right|$ 得证